**Homework 5**

**P16.1.3** Given the two functions: (a) *f*(*t*) = cos(100*πt*)sin(200*πt*) (b)  Show that  and  are periodic, determine their periods, and derive their FSEs.

**Solution:** (a) *f*(*t*) = cos(100*πt*)sin(200*πt*) = 0.5sin(100*πt*) + 0.5sin(300*πt*). The function is periodic of period 2*π*/100*π* = 0.02 s, corresponding to a frequency of 50 Hz. Its FSE consists of a fundamental and a 3rd harmonic only.

(b) =  = 0.25[0.5 – 0.5cos(200*πt*) – sin(200*πt*) + sin(400*πt*) + 0.5 + 0.5cos(600*πt*)] = 0.25 – 0.25[0.5cos(200*πt*) + sin(200*πt*)] + 0.25sin(400*πt*) + 0.125cos(600*πt*)]. The function is periodic of period 2*π*/200*π* = 0.01 s, corresponding to a frequency of 100 Hz. The FSE consists of a dc component, a fundamental, a 2nd harmonic, and a 3rd harmonic.

**P16.1.5** The current through a 1 μF capacitor is 2cos2(100*πt*) mA, where *t* is in s. Determine the period of the voltage across the capacitor.

**Solution:** *iC*(*t*) = 2cos2(100*πt*) = 1 + ; *vC*(*t*) =  contains a term that is directly proportional to *t*, so it is nonperiodic.

**P16.1.8** Given *f*(*t*) = 2cos(100*πt*) + 3cos(300*πt*) + 6cos(500*πt*) + 9sin(300*πt*). Determine the coefficients of the exponential FSE of *f*(*t*).

**Solution:** *Cn* = (*an* – *jbn*)/2 and *C-n* = (*an* + *jbn*)/2. For the fundamental, *an* = 2, *bn* = 0. Hence, *C*1 = *C*-1 = 1. For the third-harmonic, *a3* = 2, *bn* = 9. Hence, *C3* = 1.5 – *j*4.5, *C-*3 = 1.5 + *j*4.5. For the fifth-harmonic, *a*5 = 6, *bn* = 0. Hence, *C*5 = *C*-5 = 3.

**P16.1.9** Specify the type of symmetry of the periodic function shown in Figure P16.1.9 and characterize the coefficients *an* and *bn* of its FSE.



**Solution:** The function is quarter-wave symmetric and odd; hence, *an* = 0 for all *n*, *bn* = 0 for even *n* and nonzero for odd *n*.

**P16.1.14** Derive the FSE expansion of the periodic function shown in Figure P16.1.14.



**Solution:** *f*(*t*) = 8*t*, 0 ≤ *t* ≤ 1 and *f*(*t*) = 8, 1 ≤ *t* ≤ 2, *a*0 = (4+ 8) = 6; *an* =  =  +

=  ; *ω*0 = , hence: *an* = (cos*nπ* - 1); *an* = , *n* = 1, 3, 5, …

*bn* =  = -

, *n* = 1, 2, 3,…

*f*(*t*) = 6 −cos*nπt* −sin*nπt.*

**P16.1.21** The periodic function shown in Figure P16.1.21 is described by: *f*(*t*) = 3 + sin*t*, 0 ≤ *t* ≤ *π*, and *f*(*t*) = -2 – sin*t*, *π* ≤ *t* ≤ 2*π*. Determine the average value of *f*(*t*) and the fundamental component.



**Solution:**  = == = 0.5 + 2/*π* = 1.137. Note that this is consistent with *f*(*t*) being the sum of a full-wave rectified waveform of amplitude 1 and a square wave of amplitude 3 in the positive direction and of amplitude -2 in the negative direction. The average of this square waveform is (3×*π* – 2×*π*)/2*π* = 0.5, and the average of the full-wave rectified waveform is 2/*π.*

The function is neither odd nor even nor half-wave symmetric. *ω*0 = 1 rad/sand *T* = 2*π*.    . Neglecting the sine terms, *a*1 = 0. This is consistent with *f*(*t*) being the sum of a full-wave rectified waveform and a square wave, since the FSE of the full-wave waveform does not have a fundamental at the frequency of the sine or cosine waveform which it is derived (Equation 16.4.4).

 . Neglecting the sine terms, *b*1 =  . The fundamental component is therefore 3.183sin*t*. This is the same as the fundamental of a square wave whose peak-to-peak amplitude is 5 (Equation 16.2.28 with *Am* = 5/2 = 2.5). The full-wave rectified waveform does not have an ac component of period *T*, the lowest frequency being 2/*T*.

Alternatively, the function may be advanced or delayed by a quarter of a period, in which case it becomes an even function. If advanced by *π*/2, the function becomes as shown. The FSE of this function has a dc component, as derived above, but its ac component has cosine terms only. It follows that:

 . The fundamental of *fad*(*t*) is . When delayed by *π*/2, the phase angle becomes (*t* – *π*/2), and the fundamental becomes , as before.



**P16.1.28** Derive the FSE of the waveform of Figure

P16.1.28 in two ways: (a) direct evaluation

of coefficients; (b) from that of its derivative.

**Solution:** (a) *a*0 = *C*0 = ;  + = = =  = .



(b) When differentiated, the result is the rectangular pulse train shown, of zero average. Compared to the pulse train of Figure 16.2.7, it has *A* = 4, *α* = 1/4, and is advanced by



1/2 s. Hence, . The integral waveform has    = , as above. The constant of integration is the average value of the integrated function.

**P16.1.31** A periodic function of period 1 s is defined as , . Determine how the magnitudes of the harmonics vary with the order *n* of the harmonic.



**Solution:** *f*(*t*) is an odd function, as shown, since . It is continuous at *t* = +0.5 and *t* =

-0.5, where . , which is even, as shown. It is continuous at *t* = +0.5 and *t* = -0.5, where 

, which is odd, as shown, and is discontinuous at the beginning and end of each period. Since the second derivative is the first discontinuous derivative, the harmonics vary with *n* as 1/*n*3.